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# Optimization of green-times at an isolated urban crossroads

M.E. Fouladvand[1,3,a] and M. Nematollahi[2]

[1] Department of Physics, Zanjan University, PO Box 313, Zanjan, Iran
[2] Tehran Traffic Control Company (TTCC), PO Box 15836, Tehran, Iran
[3] Institute for Studies in Theoretical Physics and Mathematics, PO Box 19395-5531, Tehran, Iran

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**Abstract.** We propose a model for the intersection of two urban streets. The traffic status of the crossroads is controlled by a set of traffic lights which periodically switch to red and green with a total period of $T$. Two different types of crossroads are discussed. The first one describes the intersection of two one-way streets, while the second type models the intersection of a two-way street with an one-way street. We assume that the vehicles approach the crossroads with constant rates in time which are taken as the model parameters. We optimize the traffic flow at the crossroads by minimizing the total waiting time of the vehicles per cycle of the traffic light. This leads to the determination of the optimum green-time allocated to each phase.

**PACS.** 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 82.20.Mj Nonequilibrium kinetics – 45.70.Vn Granular models of complex systems; traffic flow

## 1 Introduction

Over the past decade, the vehicular traffic problems has been intensively investigated within the context of statistical physics (for a review see Refs. [1–4]). Modeled as a system of *interacting* particles driven far from equilibrium, vehicular traffic presents the possibility to study various aspects of truly non-equilibrium systems which are of current interest in statistical physics [5–7]. The majority of these studies have been allocated to the *highway traffic*. The other area in vehicular traffic is *urban traffic* which have also been studied by statistical physicists [1]. The simulation of traffic flow in a large-sized city is a formidable task and many degrees of freedom have to be involved (see *e.g.* [8–10]). In practice, streets of a city form a network of junctions that are linked together. Each crossroads receives demands (vehicles attempting to pass the cross) and at each crossroads there exists a traffic light which, with some certain programming, controls the transportation. The first model introduced by statistical physicists for the description of the city network, known as the BML model in the literature, uses a deterministic cellular automata framework [11] and predicts a sort of phase transition from free-flow to a jammed state. In the model, each site of a square lattice represents the crossing of a single-lane east-west street and a single-lane north-south street (no turning of vehicles are allowed). The state of east-bound vehicles are updated synchronously at every odd discrete time steps whereas those of the north-bound vehicles are updated in parallel at every even time step following a rule which is simple extension of totally asymmetric simple exclusion process (TASEP) [6,7].

The BML model has been generalized to take into account several realistic features of traffic in cities such as asymmetric distribution of cars [12], faulty traffic lights [13], independent turning of the vehicles [14,15] and green-wave synchronization [16]. At first sight, the BML model and the above-mentioned extensions seem unrealistic because the vehicles hop from one crossing to the next. In a more realistic generalization, each connecting bond of neighbouring junctions were replaced by a *decorated bond* [17–19]. Later, a more serious model of city network was introduced by Chowdhury and Schadschneider (CS model) [20] which developed a more detailed *fine-grained* description of city traffic. This model combines the BML mode together with the Nagel-Schreckenberg model of highway traffic [21,22]. In the CS model, the signals are synchronized in such a way that all the signals remain green for the east-bound vehicles for a time interval $T$ and then, simultaneously turn red for the east-bound vehicles. To the best of our knowledge, The CS model and its generalization [23] is the most realistic model introduced by the statistical physicists for city network but nevertheless, despite its nice formulation, there are still a lot of simplifications which prevent it from being an *effective* and applicable model to a city network.

In spite of developing models for describing the city network, no detailed description of an *isolated crossroads* has yet been explored. We strongly believe that in order to have a better insight to the problem of city network, one must have a clear picture at single crossroads which

[a] e-mail: `foolad@theory.ipm.ac.ir`

would definitely be of great importance for an optimal programming of the entire network. The empirical mechanism by which the traffic lights are controlled is generally divided into two distinct methods: fixed time and real-time. In the fixed-time method, a fixed value of time is allocated to the traffic light as well as its sub-phase times. In the real-time method, which is becoming increasingly popular in great cities, the ensemble of crossroads are intelligently controlled by a central controller. The control mechanism is usually based on the concept of producing a kind of *green waves* between the crossroads. These waves interact with each other, and if the passing-demands are of high values, the green waves may have destructive effects on each other, and hence, the concept of green wave may fail to be the final solution for optimizing the overall flow. Knowing the local optimum behaviour of single intersections could give us an appropriate criterion to control the set of crossroads in an adaptive manner. Additionally, not all the intersections of a city are highly affected by the neighbouring intersections. To a good approximation, marginal intersections could be regarded as isolated crossroads and therefore single-crossroads optimization strategies need to be investigated. In this paper we aim to analyze a single crossroads in detail in order to find a better insight to the problem of optimizing the total flow in cities.

The organization of this paper is as follows: In Section 2, we introduce the model, state our strategy for optimizing the traffic flow at the *one-way to one-way* intersection and finally obtain the optimum green-times of the corresponding phases. In Section 3, we extent our model to allow for a three-phase *one-way to two-way* crossroads. Section 4 is devoted to some empirical data on two of the Tehran crossroads which are under the control of an *intelligent traffic controller system.* We compare the averaged green-time proposed by system to the optimum green-time of our fixed time theory. Our theory input-parameters are obtained *via* the empirical data. Finally we give our concluding remarks in Section 5.

## 2 Formulation of the model

### 2.1 One-way to one-way crossroads

Let us consider a single crossroads which is the result of the intersection of two perpendicular streets. In their simplest structure, these streets can each direct a one-way traffic flow. With no loss of generality, we take them as one-way South to North (S-N) and West to East (W-E) streets. Cars arrive at the south and the west entrances of the crossroads. In our model, we assume that the arrival rates of the cars, *i.e.*, the number of cars reaching the crossroads per second, are constant in time. Although everyday driving experiences in cities indicates that these rates have inevitable fluctuations in the course of time, yet in definite time intervals, the assumption of the constant arrival rates could be justified at least on an average level. As will be seen in what follows, this assumption leads to great simplifications. We take the arrival rates to be $\alpha_1$ (for S-N cars) and $\alpha_2$ (for the W-E cars) respectively. Also we denote the passing-rate of cars (number of cars passing the crossroads in the unit of time during the green-phase) by $\beta_1$ and $\beta_2$. The period of the traffic lights is taken to be a definite value $T$ which is assumed to remain constant. The starting time of each cycle of the traffic light is the moment at which the light turns green for the S-N street. The S-N light remains green for $T_1$ seconds. At $T_1$ the traffic lights turn red for the S-N street and simultaneously changes to green for the W-E street. This is the beginning of the second phase which continues from $T_1$ to $T$ (end of the cycle). During Phase I ($0 \le t \le T_1$), the S-N cars can pass the crossroads northwards and W-E cars are stopped for the red light. Over Phase II ($T_1 \le t \le T$), the S-N cars must wait behind the red light whilst the W-E cars are eastwards passing the crossroads.

Now the basic question is "*how should traffic engineers adjust the value of $T_1$ in order to optimize the traffic flow through the intersection*?". With the assumption that the number of passengers in each car takes an equal average value for each direction, the optimization task is realized by minimizing the total waiting time of the cars per cycle of the traffic light. For this purpose, we introduce two quantities $N_1$ and $N_2$ which represent the number of cars stopping (queues) at the red lights in the red phases of the S-N and W-E streets respectively. Clearly $N_1$ and $N_2$ are functions of time and, in general, are divided into different lanes on the streets. The dynamics of $N_1$ and $N_2$ are read from the following equations in which $n$ denotes the cycle number of the traffic light:

$$N_1(nT+T_1) = [N_1(nT) + (\alpha_1 - \beta_1)T_1]\theta \tag{1}$$

$$N_2(nT+T_1) = N_2(nT) + \alpha_2 T_1 \tag{2}$$

$$N_1((n+1)T) = N_1(nT+T_1) + \alpha_1(T-T_1) \tag{3}$$

$$N_2((n+1)T) = [N_2(nT+T_1) + (\alpha_2 - \beta_2)(T-T_1)]\theta. \tag{4}$$

The $\theta$ symbols ensure the positiveness of the quantities in the brackets, *i.e.*, the value of $\theta$ is one if the quantity in the bracket is positive and zero elsewhere. This limitation is dictated to us since, by definition, the quantities $N_1$ and $N_2$ can only take positive values (they denote queues' lengths). Let us investigate the different situations in more details. For instance in the equation (1), the case $\theta = 1$ which corresponds to $N_1(nT+T_1) > 0$ describes the situation that after the S-N lights goes red (in the $n$th cycle), the whole queue of vehicles has not pass the cross and only a part of the queue has managed to pass during the green-phase. The other case, *i.e.*, $\theta = 0$ which corresponds to $N_1(nT+T_1) = 0$ indicates that the $n$th queue waiting in S-N direction has completely passed the cross during the time interval $nT \le t \le nT+T_1$. The same arguments apply to equation (4).

We now define the total waiting time (TWT) of the vehicles per cycle of the traffic light. It is the total time wasted by the vehicles during their stop in the red phases. The TWT is the sum of the sub-waiting times of each direction. Denoting the TWT and the sub waiting times by

$T^w$, $T^{(w,1)}$ and $T^{(w,2)}$ respectively, the following equations could be written for the $n$th cycle.

$$T^{(w,1)}_{n\to n+1} = N_1(nT+T_1)(T-T_1) + \frac{1}{2}\alpha_1(T-T_1)^2 \quad (5)$$

$$T^{(w,2)}_{n\to n+1} = N_2(nT)T_1 + \frac{1}{2}\alpha_2 T_1^2. \quad (6)$$

Let us consider equation (5). The waiting time of the $(n+1)$th S-N queue is divided into two part. The first part is related to the initial length of the queue: $N_1(nT+T_1)$. If this initial part has a non-zero length, then the time wasted by the initial vehicles is simply their number times the total period of the red phase. This leads to the first term of equation (5). The second part is related to the contribution given by the new oncoming vehicles arriving at the S-N direction of the crossroads during the red period which lasts for $T-T_1$ seconds. Since we have assumed the vehicles arrive at a constant rate $\alpha_1$, the time wasted by the vehicles arriving in the infinitesimal interval $[t, t+\mathrm{d}t]$ of the red interval is simply their number $(\alpha_1 dt)$ times the remaining time to the green signal $(T-T_1-t)$. Therefore the total contribution is given by integrating over the red period:

$$\int_0^{T-T_1} \alpha_1 \mathrm{d}t(T-T_1-t) = \frac{1}{2}\alpha_1(T-T_1)^2$$

which is the second term on the right hand side of equation (5). Similar arguments lead the equation (6).

As clearly can be seen, the analytical expression of the TWT strongly depends on the positiveness of the queues' lengths just after the lights go red. In what follows, we show that different traffic status can be identified according to the behaviour of the quantities $N_1(nT+T_1)$ and $N_2(nT)$. Let us look at the first cycle, *i.e.*, $n=0$. If $N_2(0)=0$ (a complete passing of the previous W-E queue) then it is easily seen that in order to have a complete passing of the next W-E queue, one should have the condition $\alpha_2 T - \beta_2(T-T_1) \le 0$. In this case, one has $N_2(T)=0$. It can be easily verified that that provided that the above stability condition holds, we have no W-E queues after the W-E light goes red for the general $n$th cycle. This characterizes a *light traffic* state in which the whole queue can pass the crossroads during one green time. Similar arguments for the S-N direction shows that provided the stability condition $\alpha_1 T - \beta_1 T_1 \le 0$ holds, we have a stable, light traffic state in the S-N direction and that $N_1(nT+T_1)=0$ for general $n$. The case which traffic condition is light in both directions (State I) is characterized by the following stability conditions:

$$\alpha_2 T - \beta_2(T-T_1) \le 0\ ; \quad \alpha_1 T - \beta_1 T_1 \le 0 \quad (7)$$

which result in the following relations:

$$N_1(nT+T_1) = N_2(nT) = 0, \quad n = 1,2,... \quad (8)$$

In this state, The TWT is independent of the cycle number $n$. Putting the above conditions into equations (5, 6) and minimizing the TWT with respect to $T_1$ leads to the following equation:

$$T_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} T. \quad (9)$$

Inserting the above answer in the stability conditions (7) yields to the following constraints among the rates:

$$\beta_2 \ge \alpha_1+\alpha_2, \quad \beta_1 \ge \alpha_1+\alpha_2. \quad (10)$$

We now investigate a totally different situation, *i.e.*, a crowded crossroads in both directions. Let us again look at the first cycle. Supposing the first cycle is characterized by the conditions $N_2(0), N_1(T_1) > 0$. one could easily verify that provided the following relations hold:

$$\alpha_1 > \beta_1, \quad \alpha_2 T > \beta_2(T-T_1). \quad (11)$$

We have a stable condition in the next cycles:

$$N_1(nT+T_1), \quad N_2(nT) > 0. \quad (12)$$

In sharp contrast to the State I, in this state which is characterized by the equation (11) and referred to as the state II, the values of $N_1$ and $N_2$ are functions of the cycle number. This is easily seen by the following relations:

$$N_1(nT) = N_1(0) + n(\alpha_1 T - \beta_1 T_1) \quad (13)$$

$$N_2(nT) = N_2(0) + n[\alpha_2 T - \beta_2(T-T_1)] \quad (14)$$

$$N_1(nT+T_1) = N_1(0) + n(\alpha_1 T - \beta_1 T_1) + (\alpha_1-\beta_1)T_1 \quad (15)$$

$$N_2(nT+T_1) = N_2(0) + n[\alpha_2 T - \beta_2(T-T_1)] + \alpha_2 T_1. \quad (16)$$

The above relations show that queues' lengths grow linearly with time and a complete passing of a queue in one cycle is not possible. Drivers should wait more than one cycle in order to pass the crossroads. It can be shown that in the large cycle-number limit, the sub-waiting times are as follows:

$$T^{(w,1)}_{n\to n+1} \sim n(T-T_1)(\alpha_1 T - \beta_1 T_1) \quad (17)$$

and

$$T^{(w,2)}_{n\to n+1} \sim nT_1(\alpha_2 T - \beta_2(T-T_1)). \quad (18)$$

We now minimize the TWT with respect to $T_1$ which leads to the following equation:

$$T_1 = \frac{\beta_1+\beta_2+\alpha_1-\alpha_2}{2(\beta_1+\beta_2)} T \quad (19)$$

the consistency of this solution with stability conditions (11) yields the following constraints:

$$\alpha_1 > \beta_1, \quad \alpha_1\beta_2 + 2\alpha_2\beta_1 + \alpha_2\beta_2 \ge \beta_1\beta_2 + \beta_2^2. \quad (20)$$

Also positiveness of $T_1$ itself imposes the extra restriction $\alpha_2 < \beta_1+\beta_2+\alpha_1$.

Next we consider the situation (state III) where in the first cycle of the traffic light, one has $N_1(T_1) = 0$ but $N_2(0) > 0$. This corresponds to the situation in which the S-N street has a light traffic flow while the W-E street has a heavy one. The conditions for a stable pattern are:

$$\alpha_1 T - \beta_1 T_1 \leq 0, \quad \alpha_2 T - \beta_2 (T - T_1) \geq 0. \tag{21}$$

The above stability conditions ensures the following relations:

$$N_1(nT + T_1) = 0, \qquad N_2(nT) > 0. \tag{22}$$

In the large cycle-number limit, minimizing of the TWT leads to the value $T_1 = \frac{\beta_2 - \alpha_2}{2\beta_2} T$. It could be easily checked that the above solution is inconsistent with the stability conditions, and hence, is not acceptable as an optimum signalization of traffic light. In the region determined by the stability conditions (21), the TWT is positive definite and therefore its minimum coincides with the upper limit of the inequalities (21). Therefore one finds:

$$T_1 = \max\left(\frac{\alpha_1}{\beta_1}, \frac{\beta_2 - \alpha_2}{\beta_2}\right) T. \tag{23}$$

A similar argument applies to the final state (state IV) which is characterized by $N_1(nT + T_1) > 0$ and $N_2(nT) = 0$ (it is sufficient to interchange the indices one and two).

## 3 One-way to two-way crossroads

At this stage, we consider another frequent type of a crossroads. Here we let the vehicles move in both S-N as well as N-S directions but still the vehicles in the W-E street are restricted to move eastwards. This situation describes a one-way to two-way urban intersection. Consequently each cycle consists of three phases. In the first phase which lasts for $0 \leq t \leq T_1$, the traffic light is green for the S-N cars and red for the other two directions. During the second phase which starts at $T_1$ and finishes at $T_2$, the traffic light is green for the N-S cars and red for the other two directions. In the final phase, which lasts for $T_2 \leq t \leq T$, the traffic light remains green for the W-E cars and red for the N-S as well as S-N directions. The entrance rates are taken to be $\alpha_1, \alpha_2$ and $\alpha_3$ and we denote the passing rate by $\beta_1, \beta_2$ and $\beta_3$ for each direction respectively. The starting time of the cycles is chosen to be the moment at which the traffic light turns green for the S-N direction. Similar equations for the queues' lengths $N_1, N_2$ and $N_3$ could be written down and in principle one can evaluate the TWT in terms of these quantities. The exact form of the TWT strongly depends on the positiveness of the queues' lengths just after the traffic lights goes red for the respective direction. In the case under consideration, eight different possibilities are identified due to the traffic conditions (two possibilities for each movement direction). Now we write the explicit expression for the TWT during the $n \to n+1$ cycle.

$$T^{(w,1)}_{n\to n+1} = N_1(nT + T_1)(T_2 - T_1) + \frac{1}{2}\alpha_1 (T - T_1)^2 + N_1(nT + T_2)(T - T_2) \tag{24}$$

$$T^{(w,2)}_{n\to n+1} = N_2(nT)T_1 + \frac{1}{2}\alpha_2 T_1^2 + N_2(nT + T_2)(T - T_2) + \frac{1}{2}\alpha_2 (T - T_2)^2 + \alpha_2 T_1 (T - T_2) \tag{25}$$

$$T^{(w,3)}_{n\to n+1} = N_3(nT)T_1 + \frac{1}{2}\alpha_3 T_2^2 + N_3(nT + T_1)(T_2 - T_1). \tag{26}$$

Let us only discuss the most probable one which corresponds to light traffic conditions in all directions:

$$N_1(nT + T_1) = N_2(nT + T_2) = N_3(nT) = 0. \tag{27}$$

It could be easily verified that the triple stability condition for the validity of the above assumptions are as follows:

$$\alpha_1 T - \beta_1 T_1 \leq 0, \quad \alpha_2 T - \beta_2 (T_2 - T_1) \leq 0 \quad \text{and} \quad \alpha_3 T - \beta_3 (T - T_2). \tag{28}$$

Inserting equation (27) into equations (24–26) and minimizing the TWT with respect to $T_1$ and $T_2$ leads to the following fixation of $T_1, T_2$.

$$T_1 = T\frac{\alpha_1(\alpha_2 + \alpha_3) - \alpha_2\alpha_3}{\alpha_1(\alpha_2 + \alpha_3) + \alpha_2\alpha_3} \tag{29}$$

$$T_2 = T\frac{2\alpha_1\alpha_2}{\alpha_1(\alpha_2 + \alpha_3) + \alpha_2\alpha_3}\cdot \tag{30}$$

One directly observes that in the symmetric case of equal arrival rates ($\alpha_1 = \alpha_2 = \alpha_3$), $T_1$ and $T_2$ take the expected values $\frac{T}{3}$ and $\frac{2T}{3}$ respectively. The other point which must be mentioned is that in this traffic state, $T_1$ and $T_2$ do not depend on the passing rates, and are solely determined by the arrival rates.
Another extreme is the situation when all of the three directions are carrying a heavy traffic flow. It could be anticipated that the stability conditions for the positiveness of $N_1(nT+T_1), N_2(nT+T_2)$ and $N_3(nT)$ are $\alpha_1 > \beta_1$, $\alpha_2 T_2 - \beta_2(T_2 - T_1) > 0$ and $\alpha_3 T - \beta_3 (T - T_2) > 0$. In this case, all the three sub-waiting times depend on the cycle number $n$ and it can be shown that in the large $n$ limit, the minimization of the TWT give rises to the following values for $T_1$ and $T_2$.

$$T_2 = \frac{T}{2}\frac{2\beta_1\beta_2 + \beta_2\alpha_1 + \beta_1\alpha_2 - \alpha_3(\beta_2 + \beta_1) + \beta_3(\beta_1 + \beta_2)}{\beta_1\beta_2 + \beta_2\beta_3 + \beta_3\beta_1} \tag{31}$$

$$T_1 = \frac{T}{2}\frac{\beta_1\beta_2 + \beta_3\beta_1 + \beta_2\alpha_1 + \beta_3\alpha_1 - \alpha_2\beta_3 - \beta_2\alpha_3}{\beta_1\beta_2 + \beta_2\beta_3 + \beta_3\beta_1}\cdot \tag{32}$$

Here, in contrast to (29) and (30), the passing rates appear in the expressions of $T_1$ and $T_2$ and it can be seen that in the fully symmetric condition, one again obtains that $T_1$ and $T_2$ are one-third and two-thirds of $T$ respectively.

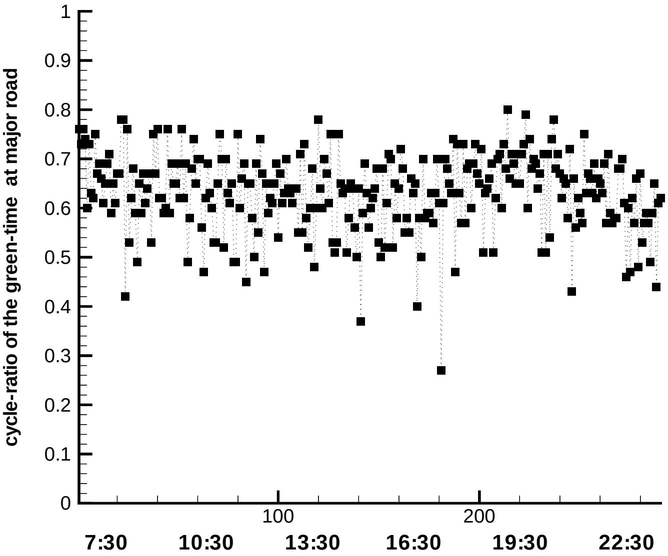


**Fig. 1.** The cycle-ratio of green-time of Abbasabad street to the total period of the traffic light cycle. The numbers on the horizontal axes are the cycle number.

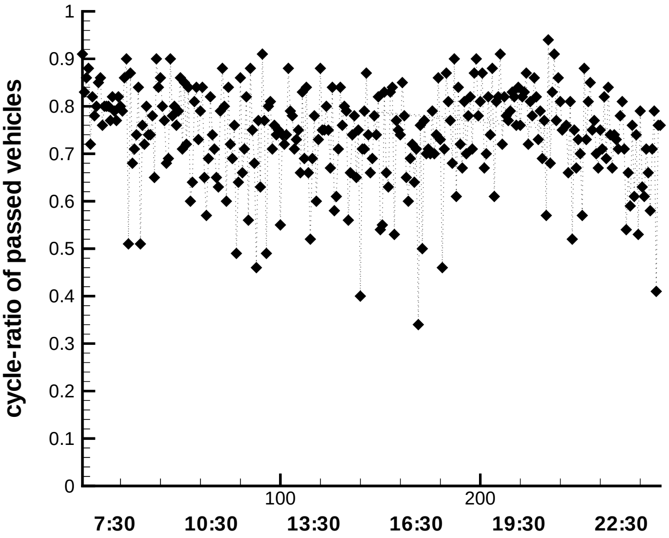


**Fig. 2.** The cycle-ratio of the passed vehicles (during the green phases) of Abassabad street to Mahnaz street of each traffic light cycle. The numbers on the horizontal axis denote the cycle number.

## 4 Empirical data

For comparison of our model to the empirical data, a time-series analysis on two of Tehran intersections were carried out. The central part of Tehran is under control of the SCATS (*Sydney Co-ordinated Adaptive Traffic System*) [24,25] that is an intelligent traffic controller. The strategy followed by most of urban traffic control systems is based on establishing green-waves along the major streets of cities. One popular strategy consists of dividing the city intersections into different sets. Each set has a leading mother crossroads (the prime crossroads in the set) and a lot of offspring crossroads. Each set is linked to the neibouring ones. The signalization of the crossroads network is determined by the implementation of green wave between adjacent sets. The allocated green times at a crossroads is proportional to the number of vehicles (traffic volume) approaching to the crossroads. Since in general, there is a natural fluctuation in the traffic volume, the amount of green times and hence the complete cycle time of traffic lights are variables in an intelligent control method.

We considered two different intersections. The first one which is located in Tehran downtown connects Valiasr St. to Takhtejamshid St. Both of these are one-way and major streets. The other crossroads connects Abbasabad St. (one-way) to Mahnaz St. (one-way). In contrast to the previous case, here the first street is a major while the second street is a minor one. The data set is provided by magnetic counting loops which are installed just before the pedestrian-lines of each crossroads. The data were collected on second of July 2000. The data consist of the sub-phase green times and the numbers of vehicles passed during the green times for each cycle of the traffic lights (traffic volumes). In our major-to-minor crossroads and for each cycle of the traffic light, we evaluated the ratio of the major street green-time to the total cycle time. We call this quantity the *time-cycle-ratio*. Similarly can can consider the number of passed vehicles, *i.e.*, traffic volume in a cycle and introduce the *volume-cycle-ratio* which for each cycle is obtained by dividing the number of passed vehicles during a sub-phase green time to the whole number of passed vehicles during a complete cycle. Figures 1–3 belong to the major-to-minor crossroads. As seen from the graphs, the time-cycle-ratio allocated to the major street strongly fluctuates due to demand fluctuations received by the crossroads. In order to have a rough estimation of our

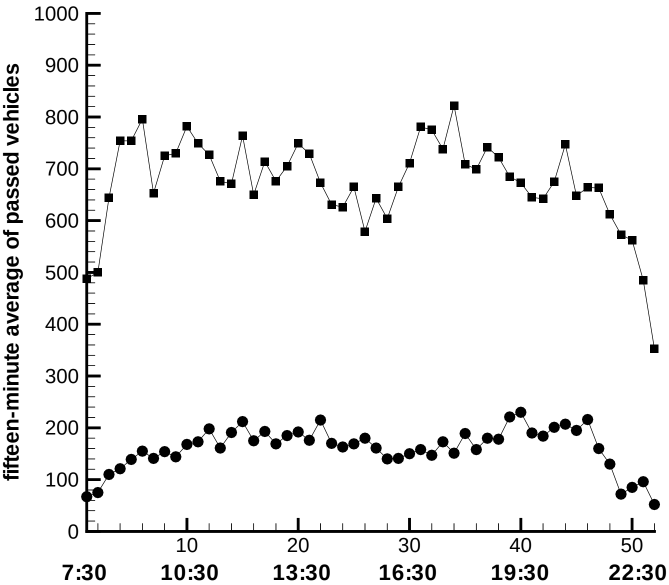


**Fig. 3.** The above graph denotes the total number of passed vehicles (from both streets) during fifteen-minute time intervals. The lower graph shows the number of passed vehicles form Mahnaz street during fifteen-minute intervals.

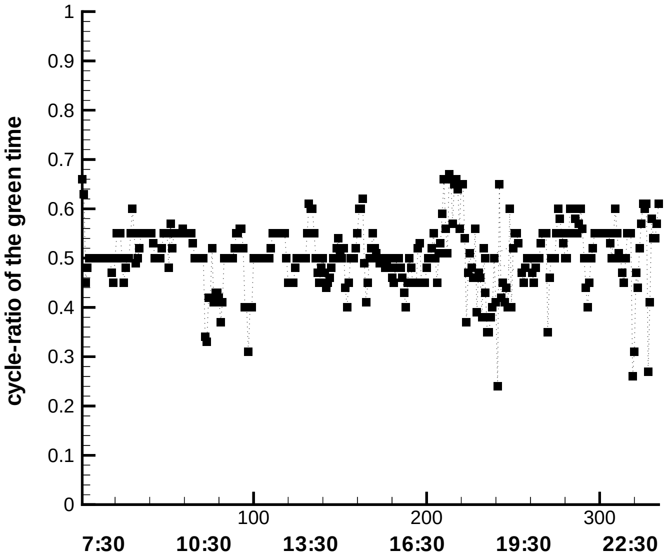


**Fig. 4.** The cycle-ratio of green time allocated to Takhtejamshid Street. The numbers on the horizontal axis denote the cycle number.

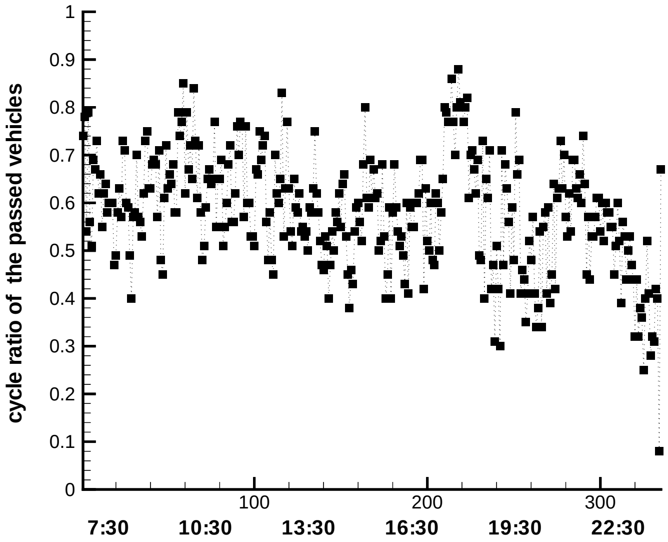


**Fig. 5.** Cycle-ratio of the passed vehicles from Takhtejamshid street to Valiasr street.

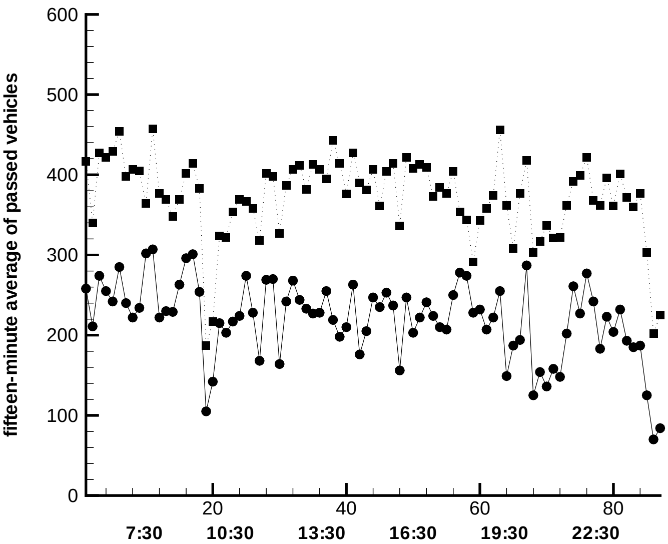


**Fig. 6.** The above graph denotes the total number of passed vehicles (from both streets) during fifteen-minute time intervals. The lower graph shows the number of passed vehicles form Takhtejamshid str. during fifteen-minute intervals.

model parameters, we considered a two-hour time interval between 12:30–14:30 during which we have the least fluctuations in the traffic volume. It is empirically observed that in this two-hour period, the traffic state is light and the queues are cleared during one cycle. Therefore, the optimal green times should be evaluated from equation (9). According to this equation, we only need to know the the ratio of $\frac{\alpha_1}{\alpha_2}$.

We should mention that due to the position of the magnetic counting loops, they are unable to measure the upstream fluxes which are directly related to the parameters $\alpha_1$ and $\alpha_2$. For a better estimation of the in-flow parameters, one should install another set of magnetic loops a few meters upwards the pedestrian-lines. We approximated the ratio of $\frac{\alpha_1}{\alpha_2}$ by the ratio of the traffic volume which has passed through the major street to the traffic volume of the minor one during the two-hour period. The two-hour traffic volumes are simply obtained by adding the cycle volumes together. This yields the value $\frac{\alpha_1}{\alpha_2} = 0.35$. Putting this value into equation (9) yields $\frac{T_1}{T} = 0.74$. On the other hand, the averaged value of the empirical time-cycle-ratio of the major street over the two-hour period leads to the result $\frac{T_1}{T} = 0.64$ which differ by ten percent from the value predicted by the theory. Figures 4–6 belong to our major-to-major crossroads. Here we focused on the interval 13:30–15:30. Analogous to the major-to-minor, the traffic state is light. The empirical averaged value of $\frac{T_1}{T}$ is 0.52 while the same amount evaluated from equation (9) is 0.56 ($T_1$ refers to the green time of Takht. Street). Here we observe that the difference between the fixed time method (theory) and real-time method (intelligent control) is less than one obtained in the major-to-minor crossroads. As depicted from the diagrams, in the major-minor crossroads, we observe more fluctuations in the time-cycle-ratio in comparison with the major-major crossroads. These fluctuations are enhanced in the volume-cycle-ratio. The least fluctuation belongs to the time-cycle-ratio of the major-major crossroads. Comparing the averaged values of these time-cycle-ratio values over certain intervals leads to a better understanding of the traffic state.

## 5 Summary and conclusion

In conclusion, we have developed a prescription for the traffic-light programming at a single urban crossroads. The method is based on minimizing the total waiting-time of cars stopping in the red phases of the traffic light. In our model the total period of the cycle is assumed to be

**Table 1.**

| Traffic state | Optimized green time | Stability conditions |
|---|---|---|
| State I (S-N light, W-E light) | $T_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} T$ | $\beta_1, \beta_2 \geq \alpha_1 + \alpha_2,$ |
| State II (S-N heavy, W-E heavy) | $T_1 = \frac{\beta_1+\beta_2+\alpha_1-\alpha_2}{2(\beta_1+\beta_2)} T$ | $\alpha_1 \geq \beta_1, \quad \alpha_1\beta_2 + 2\alpha_2\beta_1 + \alpha_2\beta_2 \geq \beta_1\beta_2 + \beta_2^2$ |
| State III (S-N light, W-E heavy) | $T_1 = \max(\frac{\alpha_1}{\beta_1}, \frac{\beta_2-\alpha_2}{\beta_2}) T$ | $\alpha_1 \leq \beta_1, \quad \beta_2 \geq \alpha_2$ |
| State IV (S-N heavy, W-E light) | $T_1 = \max(\frac{\alpha_2}{\beta_2}, \frac{\beta_1-\alpha_1}{\beta_1}) T$ | $\alpha_2 \leq \beta_2, \quad \beta_1 \geq \alpha_1$ |

a fixed value. We have also assumed that vehicles arrive at the crossroads with constant time rates. This is equivalent to a constant time-headway between cars. In reality we have a fluctuating time-headway due to the natural fluctuation in the traffic volume. As a first stage of an analytical treatment, we have taken the arrival rates to be constant in time. However, for a more realistic description, one should remove this restriction and assume that the time headway satisfies a random distribution function. Work along this assumption is in progress. The other point concerns the passing-rates. One should note that throughout the paper, the passing-rate of cars from the crossroads are taken to be constants. This is valid only if the green-phase time is not so long such that the time-headways between the cars exceed certain values. The value of $T$ should be so tuned that during the green phases, time-headway is less than a certain value. In fact, in the model, the values of the passing rate refer to the maximum capacity of cross (maximum number of cars passing the cross in the unit of time), which is plausible if the $T$ is appropriately adjusted with the congestion of the crossroads. The empirical value of the passing rates are determined by the crossroads characteristics such as road conditions, number of lanes, speed limits etc. Our model is more appropriate for rush ours during which the time-headways are minimum and the crossroads are operating with their maximum capacities. In Table 1, we have summarized our theoretical optimized green times including stability conditions in four different traffic states of the one-way to one-way crossroads.

Optimizing the traffic at each crossroads is the stating point of the more comprehensive problem of city network. Nevertheless, our model best suits those marginal intersections of cities where the effect of the other crossroads is suppressed.

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